Topic 2 - Functions and Equations Workbook

Domain and Range

1. Write down the domain and range for each of the graphs shown below.

a)

b)

c)

d)

e)

minimum point is (3,-7)

f)
2. For each of the following relationships write down the domain required to transform the relationship to a function.

   a) \( f(x) = 3x - 1 \)
   b) \( f(x) = x^2 - 5x + 6 \)
   c) \( f(x) = \frac{3}{x + 1} \)
   d) \( f(x) = \frac{2}{3x} + x \)
   e) \( f(x) = x^3 \)
   f) \( f(x) = \frac{1}{x^2} \)

3. \( f(x) = x^2 + 3 \) \( g(x) = x - 4 \) \( h(x) = \frac{1}{x} \)
   a) Calculate \( f(4) \).
   b) Calculate \( g(x) = 7 \).
   c) Calculate \( h(x)^{-1} \).
   d) What is special about the inverse of \( h \)?
   e) Calculate \( g \circ f(3) \).
   f) Calculate \( g \circ h(x) = 2 \).

4. Which of the following are functions?

   a) \( f(x) = x^2 - 3x + 5 \)
   b) \( f(x) = 4 - 3x \)
   c) \( f(x) = x^2 - 3x - 28 \)
   d) \( f(x) = x^3 - 1 \)
   e) \( f(x) = (x - 2)^2 + 1 \)
   f) \( f(x) = \frac{2 - x}{x + 5} + 3 \)

5. Sketch graphs for each of each of the following relationships.

   a) \( f(x) = 4 - x \)
   b) \( f(x) = \frac{1}{x} + 3 \)
   c) \( f(x) = x^2 - 3x - 28 \)
   d) \( f(x) = x^3 - 1 \)
   e) \( f(x) = (x - 2)^2 + 1 \)
   f) \( f(x) = \frac{2 - x}{x + 5} + 3 \)

Note: All sketches on tests and exams must be accurate to 2mm and have all major characteristics clearly labelled.
Completing the Square (Part I)

1. a) Write the equation $y = x^2 - 5x + 6$ in the form $y = a(x + p)^2 + q$.
   b) Sketch the graph of the equation $y = x^2 - 5x + 6$.
   c) Write the coordinates of the minimum point of the curve.

2. a) Write the equation $y = x^2 + 3x + 2$ in the form $y = a(x + p)^2 + q$.
   b) Sketch the graph of the equation $y = x^2 + 3x + 2$.
   c) Write the coordinates of the minimum point of the curve.

3.* a) Write the equation $y = 2x^2 + 9x - 5$ in the form $y = a(x + p)^2 + q$.
   b) Sketch the graph of the equation $y = 2x^2 + 9x - 5$.
   c) Write the coordinates of the minimum point of the curve.

4. Below is a graph of $y = f(x)$.

Write the function in the form $y = a(x + p)^2 + q$, where $a$, $p$, $q$ are real numbers.
5.* $f(x)$ is a quadratic. Given that $f(x)$ has a vertex at $\left( \frac{5}{2}, -12 \right)$, and that $f\left( \frac{1}{4} \right) = -9$, write the function in the form $f(x) = a(x + p)^2 + q$, where $a$, $p$, $q$ are real numbers.

6. a) Write the equation $f(x) = x^2 - 7x + 10$ in the form $f(x) = a(x + p)^2 + q$.

b) Find the inverse of $f^{-1}(x)$.

7.* $f(x) = x^2 - 2x - 24$, find $f^{-1}(x)$.

8.** $f(x) = (3x - 2)(2x + 5)$, find $f^{-1}(x)$.

Completing the Square (Part II)

1 The graph below shows the curve of $y = a(x + p)^2 + q$.

Given that the roots of the graph above are 2 and -3, and the maximum point of the graph is $\left( \frac{5}{2}, -12 \right)$. Find the values of $p$, $q$, and $a$. 
2 A relationship is defined as $f(x) = \frac{1}{2}(x - 3)^2 - 5$.

Find,

a) the equation of the axis of symmetry,

b) the coordinates of the vertex,

c) the coordinate of the $y$-intercept,

d) the solutions to $f(x) = 0$, leaving your answers as surds.

e) the inverse of the relations, $f^{-1}(x)$.

3 A relationship is defined as $f(x) = 3x^2 - 16x + 10$.

a) Write the equation in the form $y = a(x + h)^2 + k$.

b) Hence, write down the coordinate of the vertex.

4 Write the equation of the quadratic, in the form $y = a(x + h)^2 + k$, that has a vertex at the coordinate (-3, -7) and passes through (1, 25).

5 The diagram below shows the graph of $y = a(x + h)^2 + k$.

\[ \text{Graph Image} \]

a) Give the roots of the equation.

b) Use the graph to find the value of $h$.

c) Given that $f(0) = -2$ and $f(3) = 2$ find the values of $a$ and $k$. 
1 Non calculator
Determine whether each of the following quadratics have 0, 1 or 2 solutions.

a) \( x^2 - 3x + 4 = 0 \)
b) \( 2x^2 - 7x - 1 = 0 \)
c) \( x^2 - 8x + 16 = 0 \)
d) \( 3x^2 + 8x - 5 = 1 \)
e) \( 9x^2 - 6x = -1 \)
f) \( 5x = x^2 + 4 \)

2 Non calculator
Find the value(s) of \( k \) such that the equation \( x^2 + kx + 40 = 0 \) has exactly one solution. Leave your answer(s) as a surd.

3 Non calculator
Find the set of value(s) of \( k \) such that the equation \(-kx^2 + 3x = -1 \) has two solutions.

4 Non calculator
Find the value(s) of \( k \) such that the equation \( 3x^2 + kx - 4k = 0 \) has two solutions. Leave your answer(s) as a surd.

5 Find the set of value of \( k \) such that the equation \( kx^2 + (k+1)x + 4 = 1 \) has no solutions.

6 Find the set of values of \( k \) such that the equation \( x^2 + kx + (k+5) = 0 \), has no solutions.