1 Write down the first five terms of the sequences with \(n\)th terms, \(u_n\), given for \(n \geq 1\) by

\[
\begin{align*}
a & \quad u_n = 4n + 5 \\
b & \quad u_n = (n + 1)^2 \\
c & \quad u_n = 2^n \\
d & \quad u_n = \frac{n}{n+1} \\
e & \quad u_n = n^3 - 2n \\
f & \quad u_n = 1 - \frac{1}{3}n \\
g & \quad u_n = 1 - \frac{1}{2n} \\
h & \quad u_n = 32 \times \left(\frac{1}{2}\right)^n
\end{align*}
\]

2 The \(n\)th term of each of the following sequences is given by \(u_n = an + b\), for \(n \geq 1\).
Find the values of the constants \(a\) and \(b\) in each case.

\[
\begin{align*}
a & \quad 4, 7, 10, 13, 16, \ldots \\
b & \quad 0, 7, 14, 21, 28, \ldots \\
c & \quad 16, 14, 12, 10, 8, \ldots \\
d & \quad 0.4, 1.7, 3.0, 4.3, 5.6, \ldots \\
e & \quad 100, 83, 66, 49, 32, \ldots \\
f & \quad -13, -5, 3, 11, 19, \ldots
\end{align*}
\]

3 Find a possible expression for the \(n\)th term of each of the following sequences.

\[
\begin{align*}
a & \quad 1, 6, 11, 16, 21, \ldots \\
b & \quad 3, 9, 27, 81, 243, \ldots \\
c & \quad 2, 8, 18, 32, 50, \ldots \\
d & \quad \frac{1}{2}, 1, 2, 4, 8, \ldots \\
e & \quad 22, 11, 0, -11, -22, \ldots \\
f & \quad 0, 1, 8, 27, 64, \ldots \\
g & \quad 4, 7, 12, 19, 28, \ldots \\
h & \quad \frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \frac{5}{3}, \ldots \\
i & \quad 1, 3, 7, 15, 31, \ldots
\end{align*}
\]

4 The \(n\)th term of a sequence, \(u_n\), is given by 

\[u_n = c + 3^n - 2.\]
Given that \(u_3 = 11\),

\[
\begin{align*}
a & \quad \text{find the value of the constant } c, \\
b & \quad \text{find the value of } u_6.
\end{align*}
\]

5 The \(n\)th term of a sequence, \(u_n\), is given by 

\[u_n = n(2n + k).\]
Given that \(u_6 = 2u_4 - 2\),

\[
\begin{align*}
a & \quad \text{find the value of the constant } k, \\
b & \quad \text{prove that for all values of } n, u_n - u_{n-1} = 4n + 3.
\end{align*}
\]

6 The \(n\)th term of a sequence, \(u_n\), is given by 

\[u_n = k^n - 3.\]
Given that \(u_1 + u_2 = 0\),

\[
\begin{align*}
a & \quad \text{find the two possible values of the constant } k, \\
b & \quad \text{for each value of } k \text{ found in part a, find the corresponding value of } u_5.
\end{align*}
\]

7 Write down the first four terms of each sequence.

\[
\begin{align*}
a & \quad u_n = u_{n-1} + 4, \ n > 1, \ u_1 = 3 \\
b & \quad u_n = 3u_{n-1} + 1, \ n > 1, \ u_1 = 2 \\
c & \quad u_{n+1} = 2u_n + 5, \ n > 0, \ u_1 = -2 \\
d & \quad u_n = 7 - u_{n-1}, \ n \geq 2, \ u_1 = 5 \\
e & \quad u_n = 2(5 - 2u_{n-1}), \ n > 1, \ u_1 = -1 \\
f & \quad u_n = \frac{1}{10}(u_{n-1} + 20), \ n \geq 2, \ u_1 = 10 \\
g & \quad u_{n+1} = 1 - \frac{1}{3}u_n, \ n \geq 1, \ u_1 = 6 \\
h & \quad u_{n+1} = \frac{1}{2 + u_n}, \ n \geq 1, \ u_1 = 0
\end{align*}
\]
8 In each case, write down a recurrence relation that would produce the given sequence.
   a 5, 9, 13, 17, 21, …  
   b 1, 3, 9, 27, 81, …  
   c 62, 44, 26, 8, –10, …  
   d 120, 60, 30, 15, 7.5, …  
   e 4, 9, 19, 39, 79, …  
   f 1, 3, 11, 43, 171, …

9 Given that the following sequences can be defined by recurrence relations of the form
   \( u_n = au_{n-1} + b, \ n > 1, \) find the values of the constants \( a \) and \( b \) for each sequence.
   a –4, –3, –1, 3, 11, …  
   b 0, 8, 4, 6, 5, …  
   c 3/47, 1/25, 4, 3, 1/32, …

10 For each of the following sequences, find expressions for \( u_2 \) and \( u_3 \) in terms of the constant \( k. \)
   a \( u_n = 4u_{n-1} + 3k, \ n > 1, \ u_1 = 1 \)  
   b \( u_{n+1} = ku_n + 5, \ n > 0, \ u_1 = 2 \)  
   c \( u_n = 4u_{n-1} - k, \ n > 1, \ u_1 = k \)  
   d \( u_n = 2 - ku_{n-1}, \ n \geq 2, \ u_1 = -1 \)  
   e \( u_{n-1} = \frac{u_n}{k}, \ n \geq 1, \ u_1 = 4 \)  
   f \( u_{n+1} = \sqrt[6]{61k^2 + u_n^3}, \ n > 0, \ u_1 = k\sqrt[3]{5} \)

11 A sequence is given by the recurrence relation
   \( u_n = \frac{1}{2}(k + 3u_{n-1}), \ n > 1, \ u_1 = 2. \)
   a Find an expression for \( u_3 \) in terms of the constant \( k. \)
   Given that \( u_3 = 7, \)
   b find the value of \( k \) and the value of \( u_4. \)

12 For the sequences given by the following recurrence relations, find \( u_4 \) and \( u_1. \)
   a \( u_n = 3u_{n-1} - 2, \ n > 1, \ u_3 = 10 \)  
   b \( u_{n+1} = \frac{3}{4}u_n + 2, \ n > 0, \ u_3 = 5 \)  
   c \( u_{n-1} = 0.2(1 - u_n), \ n > 0, \ u_3 = -0.2 \)  
   d \( u_n = \frac{1}{2}\sqrt{u_{n-1}}, \ n > 1, \ u_3 = 1 \)

13 A sequence is defined by
   \( u_{n-1} = u_n + c, \ n \geq 1, \ u_1 = 2, \)
   where \( c \) is a constant. Given that \( u_5 = 30, \)
   a the value of \( c, \)
   b an expression for \( u_n \) in terms of \( n. \)

14 The terms of a sequence \( u_1, u_2, u_3, \ldots \) are given by
   \( u_n = 3(u_{n-1} - k), \ n > 1, \)
   where \( k \) is a constant. Given that \( u_1 = -4, \)
   a find expressions for \( u_2 \) and \( u_3 \) in terms of \( k. \)
   Given also that \( u_3 = 7u_2 + 3, \)
   b the value of \( k, \)
   c the value of \( u_4. \)

15 A sequence of terms \( \{t_n\} \) is defined, for \( n > 1, \) by the recurrence relation
   \( t_n = kt_{n-1} + 2, \)
   where \( k \) is a constant. Given that \( t_1 = 1.5, \)
   a find expressions for \( t_2 \) and \( t_3 \) in terms of \( k. \)
   Given also that \( t_3 = 12, \)
   b find the possible values of \( k. \)

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