Part A: Quadratic Modeling in Sport

You have seen that quadratic functions can be written in different ways.

- \( y = ax^2 + bx + c \)
- \( y = a(x-h)^2 + k \)
- \( y = a(x-p)(x-q) \)

1. For each equation, explain what information you can obtain about the graph from the different letters (parameters).

2. Modeling Problem 1: Tiger in Trouble

You are to find a curve that can model a golf shot. The shot must satisfy the following criteria:

- The golfer plays the shot from a coordinate of \((0, 0)\).
- The ball must clear a tree which is 30 metres away from the golfer and which is 20 metres high.
- The shot should be played such that the ball stays as low as possible but clears the tree.

Decide which form of the quadratic function is going to be most useful. To help you decide you should consider the following questions.

- Which information needs to be “fixed”?  
- Which equation will enable you to fix this information?

Once you have decided which form of equation is most suitable you should fix the parameters you need and then find suitable values for the other parameters. Draw a graph of your curve and show that it satisfies the given requirements.

Use your model to answer the following question.
The original shot was played from a distance of 125 metres from the hole. After playing the shot over the tree how far is the ball from the hole?
3. Modeling Problem: Six in Cricket

You are to find a curve that can model a cricket shot. The shot must satisfy the following criteria:

- The batsman plays the shot from a coordinate of (0, 0)
- The ball must clear the boundary rope that is 75m away from the batsman.

Decide which form of the quadratic function is going to be most useful. To help you decide you should consider the following questions.

- Which information needs to be “fixed”?
- Which equation will enable you to fix this information?

Once you have decided which form of equation is most suitable you should fix the parameters you need and then find suitable values for the other parameters.

Find three curves that could be used to model the shot. Check that each curve fits the required criteria.

Decide which of the curves would be most appropriate and give reasons for your choice.

Use your model to answer the following question.
A fielder is able to jump to a height of 2.5 metres to catch a ball. How far inside the boundary rope should he stand if he is to catch a ball that follows your model?
4. Modeling Problem: Basketball Free Throw

You are to find a curve that can model a free throw in basketball. The throw must satisfy the following criteria:
- The thrower stands 15ft away from the basket
- The thrower releases the ball from a height of 8ft
- The basket is 10ft above the ground.

Decide which form of the quadratic function is going to be most useful. To help you decide you should consider the following questions.
- Which information needs to be “fixed”?
- Which equation will enable you to fix this information?

Once you have decided which form of equation is most suitable you should fix the parameters you need and then find suitable values for the other parameters.

Find three curves that could be used to model the shot. Check that each curve fits the required criteria.

Decide which of the curves would be most appropriate and give reasons for your choice. Comment on any problems that the rejected models may encounter.

Use your model to answer the following question.
Suppose an opposing player is able to stand 5ft away from the player making the throw and is able to block shots up to a height of 10ft. Check if your model will allow the opposing player to block the shot. If the shot will be blocked find another model that will allow the player making the throw to score a successful basket.
Part B: Modeling Questions

5. For the first 8 minutes of a chemical reaction, the mass $m$ grams of a substance $A$ is given by the quadratic formula $m = at^2 + bt$, where $a$ and $b$ are constants and $t$ (in minutes) is the time after the start of the reaction. The reaction also involves a second substance $B$.

A table of values for values of the mass of the substance $A$ at time $t$ is given below.

<table>
<thead>
<tr>
<th>Time ($t$)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of substance ($m$)</td>
<td>0</td>
<td>0.3</td>
<td>$p$</td>
<td>2.1</td>
<td>3.6</td>
<td>6.3</td>
<td>7.8</td>
<td>$q$</td>
<td>13.6</td>
</tr>
</tbody>
</table>

A. Using Geogebra, plot points that show the information for $m$ against $t$.

B. One value of $m$ has been wrongly measured. Which value is it? What should it be?

C. Find the formula for $m$ in terms of $t$, using the quadratic model.

D. Use the formula to predict the values of $p$ and $q$.

E. Draw a tangent to the graph at $t = 4$ and use it to find the instantaneous rate of change in the mass after 4 minutes.

F. The second substance $B$ has mass $M$ grams where $M = 14 - m$. On the same graph, plot points that show the graph of $M$ against $t$.

G. Write down a formula for $M$ in terms of $t$.

H. For what value of $t$ do the two substances have the same mass?

I. The introduction of a catalyst is known to speed up the reaction so that it is completed in exactly half the time. Use this information to change the formula for the mass $m$ of substance $A$ when the catalyst is present in the reaction.
A ball is thrown from a point $A$, one metre above the ground, towards a wall.

The ball travels from $A$ to $B$ along a path given by the equation $y = a + bx - x^2$.

The horizontal ground is shown by the $x$–axis and the wall is shown by the $y$–axis.

The ball passes through the point $T(2, 4)$ and hits the wall $4$ m above $O$ at $B$.

A Use the information given about the path of the ball from $A$ to $B$ through $T$ to find the values of $a$ and $b$ and hence find the equation of the path of the ball from $A$ to $B$.

B Use the equation you have found to calculate the $x$–coordinate of $A$.

C The ball rebounds from the wall at $B$, hitting the ground at $G$. The equation of the path of the ball from $B$ to $G$ is given by $y = c - 2x - x^2$. Find the value of $c$.

D Find the position of the ball when it hits the ground at $G$.

E Find the greatest height of the ball during its motion.
A pottery makes vases with a cross section as shown in the diagram to the right.

The sections $AB$, $CD$, and $EF$ are straight lines. $BC$ and $EF$ are two parts of the same quadratic function.

The points $A$, $B$, $E$, and $F$ have coordinates as follows:

$A(-4, 6.5), B(-2, 6), E(4, 6)$ and $F(6, 6.5)$

A Find the equations of the line segments $AB$ and $EF$.

B Find the equation of the line of symmetry of the vase

C The quadratic curve has equation $y = x^2 + ax + b$. Find the values of $a$ and $b$ and hence find the equation of the quadratic curve.

D Points $C$ and $D$ both lie on the $x$-axis. Find their coordinates.