1 The second and fifth terms of an arithmetic series are 40 and 121 respectively.
   a Find the first term and common difference of the series. (4)
   b Find the sum of the first 25 terms of the series. (2)

2 A sequence is defined by the recurrence relation
   \[ u_r = u_{r-1} + 4, \quad r > 1, \quad u_1 = 3. \]
   a Write down the first five terms of the sequence. (1)
   b Evaluate \( \sum_{r=1}^{20} u_r. \) (3)

3 The first three terms of an arithmetic series are \( t, (2t - 5) \) and 8.6 respectively.
   a Find the value of the constant \( t. \) (2)
   b Find the 16th term of the series. (4)
   c Find the sum of the first 20 terms of the series. (2)

4 a State the formula for the sum of the first \( n \) natural numbers. (1)
   b Find the sum of the natural numbers from 200 to 400 inclusive. (3)
   c Find the value of \( N \) for which the sum of the first \( N \) natural numbers is 4950. (3)

5 A sequence of terms \( \{u_n\} \) is defined, for \( n \geq 1 \), by the recurrence relation
   \[ u_{n+1} = k + u_n^2, \]
   where \( k \) is a non-zero constant. Given that \( u_1 = 1, \)
   a find expressions for \( u_2 \) and \( u_3 \) in terms of \( k. \) (3)
   Given also that \( u_3 = 1, \)
   b find the value of \( k, \) (3)
   c state the value of \( u_{25} \) and give a reason for your answer. (2)

6 a Find the sum of the integers between 1 and 500 that are divisible by 3. (3)
   b Evaluate \( \sum_{r=3}^{20} (5r - 1). \) (3)

7 a Prove that the sum, \( S_n, \) of the first \( n \) terms of an arithmetic series with first term \( a \) and common difference \( d \) is given by
   \[ S_n = \frac{1}{2} n[2a + (n - 1)d]. \] (4)
   b An arithmetic series has first term \(-1\) and common difference 6.
   Verify by calculation that the largest value of \( n \) for which the sum of the first \( n \) terms of the series is less than 2000 is 26. (3)

8 A sequence is defined by the recurrence relation
   \[ t_{n+1} = 4 - kt_n, \quad n > 0, \quad t_1 = -2, \]
   where \( k \) is a positive constant.
   Given that \( t_3 = 3, \) show that \( k = -1 + \frac{1}{2} \sqrt{6}. \) (6)
9  An arithmetic series has first term 6 and common difference 3.
   a Find the 20th term of the series.  \( \text{(2)} \)
   Given that the sum of the first \( n \) terms of the series is 270,
   b find the value of \( n \).  \( \text{(4)} \)

10  A sequence of terms \( t_1, t_2, t_3, \ldots \) is such that the sum of the first 30 terms is 570.
   Find the sum of the first 30 terms of the sequences defined by
   a \( u_n = 3t_n, \ n \geq 1 \),  \( \text{(2)} \)
   b \( v_n = t_n + 2, \ n \geq 1 \),  \( \text{(2)} \)
   c \( w_n = t_n + n, \ n \geq 1 \).  \( \text{(3)} \)

11  Tom's parents decide to pay him an allowance each month beginning on his 12\(^{th}\) birthday.
    The allowance is to be £40 for each of the first three months, £42 for each of the next three
    months and so on, increasing by £2 per month after each three month period.
   a Find the total amount that Tom will receive in allowances before his 14\(^{th}\) birthday.  \( \text{(4)} \)
   b Show that the total amount, in pounds, that Tom will receive in allowances in the \( n \) years
    after his 12\(^{th}\) birthday, where \( n \) is a positive integer, is given by \( 12n(4n + 39) \).  \( \text{(4)} \)

12  A sequence is defined by
    \[ u_{n+1} = u_n - 3, \ n \geq 1, \ u_1 = 80. \]
    Find the sum of the first 45 terms of this sequence.  \( \text{(3)} \)

13  The third and eighth terms of an arithmetic series are 298 and 263 respectively.
   a Find the common difference of the series.  \( \text{(3)} \)
   b Find the number of positive terms in the series.  \( \text{(4)} \)
   c Find the maximum value of \( S_n \), the sum of the first \( n \) terms of the series.  \( \text{(3)} \)

14  a Find and simplify an expression in terms of \( n \) for \( \sum_{r=1}^{n} (6r + 4) \).  \( \text{(3)} \)
   b Hence, show that
    \[ \sum_{r=n+1}^{2n} (6r + 4) = n(9n + 7). \]  \( \text{(4)} \)

15  The \( n \)th term of a sequence, \( u_n \), is given by
    \[ u_n = k^n - n. \]
    Given that \( u_2 + u_4 = 6 \) and that \( k \) is a positive constant,
   a show that \( k = \sqrt[3]{3} \),  \( \text{(5)} \)
   b show that \( u_3 = 3u_1 \).  \( \text{(3)} \)

16  The first three terms of an arithmetic series are \( (k + 4), (4k - 2) \) and \( (k^2 - 2) \) respectively,
    where \( k \) is a constant.
   a Show that \( k^2 - 7k + 6 = 0 \).  \( \text{(2)} \)
    Given also that the common difference of the series is positive,
   b find the 15th term of the series.  \( \text{(4)} \)