DP SL 11 Sequences and Series Practice Questions – Set 1

Remember that you are marked on your METHOD, ACCURACY, and ANSWER. Present all of your work as if you are submitting your final exam.

PART A:

1. (a) Show that the following sequences are arithmetic.
   (b) Find the common difference.
   (c) Define the rule that gives the nth term of the sequence.
      i. \{2, 6, 10, 14, \ldots\}  
      ii. \{20, 17, 14, 11, \ldots\} 
      iii. \{1, -4, -9, \ldots\}  
      iv. \{0.5, 1.0, 1.5, 2.0, \ldots\} 
      v. \{y + 1, y + 3, y + 5, \ldots\}  
      vi. \{x + 2, x, x - 2, \ldots\} 

2. Find the 10th term of the sequence whose first four terms are 8, 4, 0, -4.

3. Find the value of x and y in the arithmetic sequence \{5, x, 13, y, \ldots\}.

4. An arithmetic sequence has 12 as its first term and a common difference of -5. Find its 12th term.

5. An arithmetic sequence has -20 as its first term and a common difference of 3. Find its 10th term.

6. The 14th term of an arithmetic sequence is 100. If the first term is 9, find the common difference.

7. The 10th term of an arithmetic sequence is -40. If the first term is 5, find the common difference.

8. If \(n + 5, 2n + 1\) and \(4n - 3\) are three consecutive terms of an arithmetic sequences, find \(n\).

9. The first three terms of an arithmetic sequence are 1, 6, 11.
   (a) Find the 9th term.
   (b) Which term will equal 151?

10. Find \(x\) and \(y\) given that \(4 - \sqrt{3}, x, y\) and \(2 \sqrt{3} - 2\) are the first four terms of an arithmetic sequence.

11. For each of the following sequences
    (a) \(u_n = -5 + 2n, n \geq 1\)  
    (b) \(u_n = 3 + 4(n + 1), n \geq 1\)
    determine
    i. its common difference
    ii. its first term

12. The third and fifth terms of an A.P are \(x + y\) and \(x - y\) respectively. Find the 12th term.

13. The sum of the fifth term and twice the third of an arithmetic sequence equals the twelfth term. If the seventh term is 25 find an expression for the general term, \(u_n\).
PART B:

1. Find the sum of the first ten terms in the arithmetic sequences
   (a) \( \{1, 4, 7, 10, \ldots\} \)   (b) \( \{3, 9, 15, 21, \ldots\} \)   (c) \( \{10, 4, -2, \ldots\} \).

2. For the given arithmetic sequences, find the sum, \( S_n \), to the requested number of terms.
   (a) \( \{4, 3, 2, \ldots\} \) for \( n = 12 \)
   (b) \( \{4, 10, 16, \ldots\} \) for \( n = 15 \)
   (c) \( \{2.9, 3.6, 4.3, \ldots\} \) for \( n = 11 \)

3. Find the sum of the following sequences:
   (a) \( \{5, 4, 3, \ldots, -15\} \)
   (b) \( \{3, 9, 15, \ldots, 75\} \)
   (c) \( \{3, 5, 7, \ldots, 29\} \)

4. The weekly sales of washing machines from a retail store that has just opened in a new housing complex increases by 2 machines per week. In the first week of January 1995, 24 machines were sold.
   (a) How many are sold in the last week of December 1995?
   (b) How many machines did the retailer sell in 1995?
   (c) When was the 500th machine sold?

5. The fourth term of an arithmetic sequence is 5 while the sum of the first 6 terms is 10. Find the sum of the first nineteen terms.

6. Find the sum of the first 10 terms for the sequences defined by
   (a) \( u_n = -2 + 8n \)   (b) \( u_n = 1 - 4n \)

7. The sum of the first eight terms of the sequence \( \{\ln x, \ln x^2 y, \ln x^3 y^2, \ldots\} \) is given by \( 4(a \ln x + b \ln y) \). Find \( a \) and \( b \).
PART A:

1. Find the common ratio, the 5th term and the general term of the following sequences
   (a) 3, 6, 12, 24, . . . (b) 3, 1, \frac{1}{3}, \frac{1}{9}, . . . (c) 2, \frac{2}{5}, \frac{2}{25}, \frac{2}{125}, . . .
   (d) −1, 4, −16, 64, . . . (e) \frac{ab}{a}, \frac{a}{b}, \frac{a}{b^2}, . . . (f) a^2, ab, b^2, . . .

2. Find the value(s) of x if each of the following are in geometric sequence
   (a) 3, x, 48
   (b) \frac{5}{2}, x, \frac{1}{2}

3. The third and seventh terms of a geometric sequence are \frac{3}{4} and 12 respectively.
   (a) Find the 10th term.
   (b) What term is equal to 3072?

4. A rubber ball is dropped from a height of 10 m and bounces to reach \frac{5}{6} of its previous height after each rebound. Let \( u_n \) is the ball’s maximum height before its \( n \)th rebound.
   (a) Find an expression for \( u_n \).
   (b) How high will the ball bounce after its 5th rebound.
   (c) How many times has the ball bounced by the time it reaches a maximum height of \( \frac{6250}{1296} \) m.

5. The terms \( k + 4, 5k + 4, k + 20 \) are in a geometric sequence. Find the value(s) of \( k \).

6. A computer depreciates each year to 80% of its value from the previous year. When bought the computer was worth $8000.
   (a) Find its value after
      i. 3 years
      ii. 6 years
   (b) How long does it take for the computer to depreciate to a quarter of its purchase price.

7. The sum of the first and third terms of a geometric sequence is 40 while the sum of its second and fourth terms is 96. Find the sixth term of the sequence.

8. The sum of three successive terms of a geometric sequence is \( \frac{35}{2} \) while their product is 125. Find the three terms.

9. The population in a town of 40,000 increases at 3% per annum. Estimate the town’s population after 10 years.
10. Following new government funding it is expected that the unemployed workforce will decrease by 1.2% per month. Initially there are 120,000 people unemployed. How large an unemployed workforce can the government expect to report in 8 months time.

11. The cost of erecting the ground floor of a building is $44,000, for erecting the first floor it costs $46,200, to erect the second floor costs $48,510 and so on. Using this cost structure
(a) How much will it cost to erect the 5th floor?
(b) What will be to total cost of erecting a building with six floors?

PART B:

1. Find the common ratios of these geometric sequences:
   (a) 7, 21, 63, 189, . . .
   (b) 12, 4, 4/3, 4/9, . . .
   (c) 1, −1, 1, −1, 1, . . .
   (d) 9, −3, 1, 1/3, 1/9, . . .
   (e) 64, 80, 100, 125, . . .
   (f) 27, −18, 12, −8, . . .

2. Find the term indicated for each of these geometric sequences:
   (a) 11, 33, 99, 297, . . . 10th term.
   (b) 1, 0.2, 0.04, 0.008, . . . 15th term.
   (c) 9, −6, 4, −8/3 . . . 9th term.
   (d) 21, 9, 27, 81/7, . . . 6th term.
   (e) 1/3, 1/4, 3/16, 9/64, . . . 6th term.

3. Find the number of terms in each of these geometric sequences and the sum of the numbers in each sequence:
   (a) 4, 12, 36, . . ., 236196
   (b) 11, −22, 44, . . ., 704
   (c) 100, −10, 1, . . ., −10⁻⁵
   (d) 48, 36, 27, . . ., 6561/1024
   (e) 1/8, 9/32, 81/128, . . ., 6561/2048
   (f) 100, 10, 1, . . ., 10⁻¹⁰

4. Write the following in expanded form and evaluate.
   (a) \( \sum_{k=1}^{7} \left( \frac{1}{2} \right)^k \)
   (b) \( \sum_{i=1}^{6} 2^{i-4} \)
   (c) \( \sum_{j=1}^{4} \left( \frac{2}{3} \right)^j \)
   (d) \( \sum_{s=1}^{4} (-3)^s \)
   (e) \( \sum_{t=1}^{6} 2^{-t} \)

5. The third term of a geometric sequence is 36 and the tenth term is 78,732. Find the first term in the sequence and the sum of these terms.

6. A bank account offers 9% interest compounded annually. If $750 is invested in this account, find the amount in the account at the end of the twelfth year.

7. When a ball is dropped onto a flat floor, it bounces to 65% of the height from which it was dropped. If the ball is dropped from 80 cm, find the height of the fifth bounce.
8. A computer loses 30% of its value each year.
   (a) Write a formula for the value of the computer after \( n \) years.
   (b) How many years will it be before the value of the computer falls below 10% of its original value?

9. A geometric sequence has a first term of 7 and a common ratio of 1.1. How many terms must be taken before the value of the term exceeds 1000?

10. A colony of algae increases in size by 15% per week. If 10 grams of the algae are placed in a lake, find the weight of algae that will be present in the lake after 12 weeks. The lake will be considered 'seriously polluted' when there is in excess of 10000 grams of algae in the lake. How long will it be before the lake becomes seriously polluted?

11. A geometric series has nine terms, a common ratio of 2 and a sum of 3577. Find the first term.

12. A geometric series has a third term of 12, a common ratio of \( \frac{1}{2} \) and a sum of \( 32 \frac{1}{16} \). Find the number of terms in the series.

13. A geometric series has a first term of 1000, seven terms and a sum of \( 671 \frac{7}{8} \). Find the common ratio.

14. A geometric series has a third term of 300, and a sixth term of 37500. Find the common ratio and the sum of the first fourteen terms (in scientific form correct to two significant figures).

15. A $10000 loan is offered on the following terms: 12% annual interest on the outstanding debt calculated monthly. The required monthly repayment is $270. How much will still be owing after nine months.

16. As a prize for inventing the game of chess, its originator is said to have asked for one grain of wheat to be placed on the first square of the board, 2 on the second, 4 on the third, 8 on the fourth and so on until each of the 64 squares had been covered. How much wheat would have been the prize?
1. Consider the following sequences:
   Arithmetic: 100, 110, 120, 130, ...
   Geometric: 1, 2, 4, 8, 16, ...
   Prove that:
   The terms of the geometric sequence will exceed the terms of the arithmetic sequence after the 8th term.
   The sum of the terms of the geometric sequence will exceed the sum of the terms of the arithmetic after the 10th term.

2. An arithmetic series has a first term of 2 and a fifth term of 30. A geometric series has a common ratio of 0.5. The sum of the first two terms of the geometric series is the same as the second term of the arithmetic series. What is the first term of the geometric series?

3. An arithmetic series has a first term of -4 and a common difference of 1. A geometric series has a first term of 8 and a common ratio of 0.5. After how many terms does the sum of the arithmetic series exceed the sum of the geometric series?

4. The first and second terms of an arithmetic and a geometric series are the same and are equal to 12. The sum of the first two terms of the arithmetic series is four times the first term of the geometric series. Find the first term of each series. If the A.P has $d = 4$.

5. Bo-Youn and Ken are to begin a savings program. Bo-Youn saves $1 in the first week $2 in the second week, $4 in the third and so on, in geometric progression. Ken saves $10 in the first week, $15 in the second week, $20 in the third and so on, in arithmetic progression. After how many weeks will Bo-Youn have saved more than Ken?

6. Ari and Chai begin a training program. In the first week Chai will run 10km, in the second he will run 11km and in the third 12km, and so on, in arithmetic progression. Ari will run 5km in the first week and will increase his distance by 20% in each succeeding week.
   (a) When does Ari’s weekly distance first exceed Chai’s?
   (b) When does Ari’s total distance first exceed Chai’s?
Set 1 – PART A – Answers

1. i. (b) 4 (c) \( t_n = 4n - 2 \) ii. (b) -3 (c) \( t_n = -3n + 23 \) iii. (b) -5 (c) \( t_n = -5n + 6 \) iv. (b) 0.5 (c) \( t_n = 0.5n \) v. (b) 2 (c) \( t_n = y + 2n - 1 \) vi. (b) -2 (c) \( t_n = x - 2n + 4 \) 2. -28 3. 9,17 4. -43 5. 7 6. 7 7. -5 8. 0 9. (a) 41 (b) 31st 10. 2, \( \sqrt{3} \) 11. (a) i. 2 ii. -3 (b) i. 4 ii. 11 12. \( x - 8y \) 13. \( t_n = 5 + \frac{10}{3}(n-1) \)

Set 1 – PART B – Answers

1. (a) 145 (b) 300 (c) -170 2. (a) -18 (b) 690 (c) 70.4 3. (a) -105 (b) 507 (c) 224 4. (a) 126 (b) 3900 (c) 14th week 5. 855 6. (a) 420 (b) -210 7. \( a = 9, b = 7 \)

Set 2 – PART A – Answers

1. (a) \( r = 2, u_2 = 48, u_n = 3 \times 2^{n-1} \) (b) \( r = \frac{1}{3}, u_2 = \frac{1}{27}, u_n = 3 \times \left(\frac{1}{3}\right)^{n-1} \) (c) \( r = \frac{1}{5}, u_2 = \frac{2}{625}, u_n = 2 \times \left(\frac{1}{5}\right)^{n-1} \) (d) \( r = -4, u_2 = -256, u_n = -1 \times (-4)^{n-1} \) (e) \( r = \frac{a}{b}, u_2 = \frac{a}{b}, u_n = ab \times \left(\frac{1}{b}\right)^{n-1} \) (f) \( r = \frac{b}{a}, u_2 = \frac{b^2}{a^2}, u_n = a^2 \times \left(\frac{b}{a}\right)^{n-1} \) 2. (a) \( \#12 \) (b) \( \frac{\sqrt{2}}{2} \) 3. (a) \( \#96 \) (b) 15th 4. (a) \( u_n = 10 \times \left(\frac{5}{6}\right)^{n-1} \) (b) \( \frac{15625}{3888} = 4.02 \) (c) \( n = 5 \) (4 times)

5. -2, \( \frac{4}{3} \) 6. (a) \$4096 ii. \$2097.15 (b) 6.2 yrs 7. \( u_n = \frac{1000}{169} \times \left(\frac{12}{5}\right)^{n-1} \), \( 1990656 \div 4225 = 471.16 \) 8. 2,5,10 or 10,5,2.5 9. 53757 10. 108 952 11. (a) 556 156 (b) \$299 284

Set 2 – PART B – Answers

1. (a) 3 (b) \( \frac{1}{3} \) (c) -1 (d) \( -\frac{3}{1} \) (e) 1.25 (f) \( -\frac{2}{3} \) 2. (a) 216513 (b) 1.6384 \times 10^{-10} (c) \( \frac{256}{729} \) (d) \( \frac{729}{2401} \) (e) \( -\frac{81}{1024} \) 3. (a) 11; 354292 (b) 7; 473 (c) 8; 90.909090 (d) 8; 1727.788 (e) 5; 2.256 (f) 13; 111.1111111111 4. (a) \( \frac{127}{128} \) (b) \( \frac{63}{8} \) (c) \( \frac{130}{81} \) (d) 60 (e) \( \frac{63}{64} \) 5. 4; 118096 6. \$2109.50 7. 9.28 cm 8. (a) \( V_n = V_0 \times 0.7^n \) (b) 7 9. 54 10. 53.5 gms; 50 weeks 11. 7 12. 9 13. -0.5, -0.7797 14. \( r = 5, 1.8 \times 10^{10} \) 15. \$8407.35 16. \( 1.8 \times 10^{19} \) or about 200 billion tonnes.

Set 3 – Answers

1. Term 9 AP = 180, GP = 256. Sum to 11 terms AP = 1650, GP = 2047. 2. 18. 3. 12 4. 7, 12 5. 8 weeks (KEN $220 & Bo-Yen $255) 6. (a) week 8 (b) week 12 7. (a) 1.618 (b) 121379 [-121400, depends on rounding errors]