Algebra Review

Arithmetic Sequences have a common difference (increase/decrease through addition of a fixed amount).

The common difference can be found using: $d = u_{n+1} - u_n$

The n^{th} term of an arithmetic sequence is given by: $u_n = u_1 + (n-1)d$

The sum of the first n terms of an arithmetic series is given by:

$$S_n = \tfrac{n}{2}\left(u_1 + u_n\right) = \tfrac{n}{2}\left(2u_1 + \left(n-1\right)d\right)$$

Geometric Sequences have a common ratio (increase/decrease through multiplication by a fixed amount). $r = \frac{u_2}{u_1} = \frac{u_{n+1}}{u_n}$

The n^{th} term of a geometric sequence is given by: $u_n = u_1 r^{n-1}$

The sum of the first n terms of a geometric series is given by:

$$S_n = \frac{u_1\left(r^n - 1\right)}{r-1}$$

The sum of an infinite geometric series is given by:

$$S = \frac{u_1}{1-r}$$ iff r is between -1 and 1

Ex.) An arithmetic sequence has $u_{20} = 62$ and $S_{20} = 670$. Find the value of u_1 and the value of d.

Ex.) A geometric sequence has $u_1 = 2$ and a common ratio of 3. Which term number will be the first to exceed 1000 000?

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Sigma Notation

Sigma means "sum". The expression $\sum_{p=4}^{7}\left(p^2-20\right)$ means the sum of the terms for *p*-values from 4 to 7.

When there are many terms, write out the first few and look for a pattern.
For example, $\sum_{q=6}^{20}(3\times 2^q)$

Logarithms

The expression $\log_2 \frac{1}{8}$ asks, "what power needs to be applied to 2 in order to get 1/8?"

You must be able to change an equation from logarithmic to exponential form using the fact that $\log_a b = x \Leftrightarrow a^x = b$.

Ex.) Solve $\log_3 20 = x$

If no base is shown on a logarithm, the base is assumed to be 10. Natural logs have a base of *e*.

Logs are particularly useful for solving equations where the variable is in the exponent.

Ex.) Solve $5^{2x-3} = 4\times 10^{-2}$

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Log Laws

$\log ab = \log a + \log b$

$\log \frac{a}{b} = \log a - \log b$

$\log a^n = n \log a$

Other Log Facts

$10^{\log a} = a$

$e^{\ln a} = a$

$q^{\log_q a} = a$

$\log_a a = 1$

Logarithmic Equations

Short (single log) - convert to exponential form (see previous page)

Long (multiple logs) - convert to a single log of the same base on each side, drop the logs, solve, then check

ex.) $\log x + \log(x+1) = \log 2$

Recall that $\log_a a = 1$. This can be useful in long log equations that have terms without logs.

ex.) $\log_3(x) - 2 = \log_3 4$

Change of Base Rule

$$\log_b a = \frac{\log a}{\log b} = \frac{\ln a}{\ln b} = \frac{\log_c a}{\log_c b}$$

Let $\log_a 2 = x$ and $\log_a 5 = y$. Write the following in terms of *x* and *y* only.

a) $\log_a 20$

b) $\log_5 2$

Binomial Expansion

Coefficients come from Pascal's triangle:

1 1
1 2 1
1 3 3 1
1 4 6 4 1
etc.

or from combinations.

ex.) a) Expand $(2-x)^4$

b) Hence, find the coefficient of x^2 in $(3x-1)(2-x)^4$

Look for patterns in your variables to find specific terms

ex.) Find the constant term in the expansion of $\left(2x^2-\frac{1}{x}\right)^6$

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